

Reg. No.:....

Name :

First Semester B.Sc. Degree (C.B.C.S.S. – OBE-Supplementary/ Improvement) Examination, November 2024 (2019 to 2023 Admission) COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS 1C01 MAT-PH: Mathematics for Physics – I

Time: 3 Hours Max. Marks: 40

PART - A

Answer **any four** questions from among the questions 1 to 5. **Each** question carries **one** mark. (4×1=4)

- Find the nth derivative of a^{mx}.
- 2. State Lagrange's mean value theorem.
- 3. Define rank of a matrix.
- 4. If A and B are orthogonal matrices, prove that AB is also orthogonal.
- 5. Replace the polar equation $r = 4r \cos \theta$ with equivalent cartesian equation and identify the graph.

Answer any seven questions from among the questions 6 to 16. Each question carries 2 marks. (7×2=14)

6. If
$$x = \frac{1}{2} \left(t + \frac{1}{t} \right)$$
, $y = \frac{1}{2} \left(t - \frac{1}{t} \right)$, find $\frac{d^2y}{dx^2}$.

- 7. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$.
- 8. Find the nth derivative of sin³ x cos² x.

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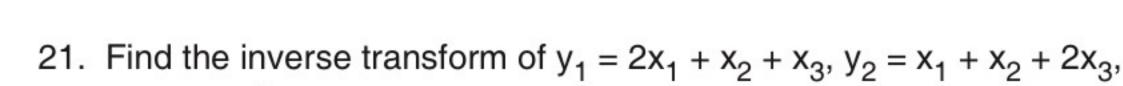
- 9. Verify Rolle's theorem for $f(x) = (x a)^m (x b)^n$ in [a, b], where m, n are positive integers.
- 10. Expand e^{sinx} by Maclaurin's series up to the term containing x⁴.
- 11. Verify Cauchy mean value theorem for the functions $f(x) = \sin x$ and $g(x) = \cos x$ in the interval [a, b].
- 12. Evaluate $\lim_{x\to 0} \frac{x \cos x \sin x}{x^2 \sin x}$.
- 13. Using the Gauss-Jordan method, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$
- 14. Solve the system of equations x + y + z = 4; x y + z = 0; 2x + y + z = 5 by determinants.
- 15. Are the vectors $x_1 = (3, 2, 7)$, $x_2 = (2, 4, 1)$ and $x_3 = (1, -2, 6)$ linearly independent. If so, find the relation between them.
- 16. For the cycloid $x = a(\theta \sin \theta)$, $y = a(1 \cos \theta)$, find $\frac{ds}{d\theta}$.

Answer any four questions from among the questions 17 to 23. Each question carries **three** marks. $(4 \times 3 = 12)$

- 17. Prove that the nth derivative of $\frac{1}{x^2 + a^2}$ is $\frac{(-1)^n n!}{a^{n+2}} \sin(n+1)\theta \sin^{n+1}\theta$. 18. If $y = e^{a\sin^{-1}x}$, prove that $(1 x^2)y_{n+2} (2n+1)xy_{n+1} (n^2 + a^2)y_n = 0$.
- 19. Expand $\log_{e}x$ in powers of (x 1) and hence evaluate $\log_{e} 1.1$ correct to 4 decimal places.
- 20. Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence







 $y_3 = x_1 - 2x_3$.

- 22. Find the radius of curvature at any point of the catenary $y = c \cosh \frac{h}{c}$.
- 23. Find a spherical co-ordinate equation for the sphere $x^2 + y^2 + (z 1)^2 = 1$.

Answer **any two** questions from among the questions **24** to **27**. **Each** question carries **five** marks. (2×5=10)

- State and prove Leibnitz's theorem for the nth derivative of the product of two functions.
- 25. Let 0 < a < b < 1. Prove that $\frac{b-a}{1+b^2} < tan^{-1}b tan^{-1}a < \frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.
- 26. Solve the equations $x_1 x_2 + x_3 + x_4 = 2$; $x_1 + x_2 x_3 + x_4 = -4$; $x_1 + x_2 + x_3 x_4 = 4$; $x_1 + x_2 + x_3 + x_4 = 0$, by finding inverse by elementary row operations.
- 27. Find the center of curvature of $x = a \cos^3 \theta$, $y = a \sin^3 \theta$.